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Visualization Approach in the Framework of APOS Learning Theory

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ABSTRACT : *The attempts to help students understand mathematics have been done either in terms of the aspect of didactic situation, pedagogical situation or pedagogical didactic situation. Visualization means the ability, process and product of a creation and interpretation by using pictures, images or diagrams to describe or communicate information, to think about or develop ideas previously unknown, or to provide further understanding. It can be considered to be an approach, which can accommodate different learning styles of students by keep following the framework of the existing theories of learning mathematics. The main idea in providing understanding of abstract mathematical knowledge is supported by the visual presentation. Then, students gradually internalize the presentation abstractly in their mind so that it will form a thinking framework ready for use when confronting problems in different contexts. Therefore, the stages of this visualization approach follow the following sequence: visualization – representation – abstraction - schematization. The sequence is an operational form (more easily applied in mathematics lesson) from the stage of Action – Process – Object – Schema in learning theory of APOS. Therefore, visualization approach proposed can be applied in the process of learning mathematics.*

KEYWORDS: *visualization, representation, abstraction, and schematization*

I. INTRODUCTION

The efforts to help students understand mathematics have been performed either in terms of the aspect of interaction student – abstract mathematical context (didactic situation), interaction teacher – student and student – student (pedagogical situation), or interaction teacher – student – mathematical content (pedagogical didactic situation). We know the realistic approach of mathematics that brings the abstract mathematics into realistic life of students (Freudenthal, 2002), cooperative learning of *jigsaw*, which focuses on the creation of interactions between interdependent groups of students (Aranson in Walker&Crogan, 1998), and indirect approach to enhance the ability of high-level mathematical thinking by creating stimulation of interaction teacher – student – mathematical content through indirect intervention (Suryadi, 2005).

On the other hand, we realize that learning style of students, in terms of the options how students "receive and process information," is different from one to another. Felder & Silverman (1998) indentified five models of student's learning style, namely (1) *Sensory to intuitive* that turns from learning through concrete objects to learning abstractly, (2) *Visual to auditory* that turns from learning through visual representation to learning through verbal explanations, (3) *Inductive to deductive* that shifts from learning started from examples to learning using general principals, (4) *Active to reflective* that shifts from learning by doing an activity to learning through thinking and learning independently, and (5) *Sequential to global* that turns from learning through linear thinking process to holistic thinking.

Although student's learning style is different from one to another, but the process of constructing understanding of mathematical knowledge should follow the existing learning theories. Dubinsky & McDonald (2001) provide the stages on how someone processes information until the formation of meaningful mathematical concept: *Action – Process – Object – Schema*. Actionis a transformation of object perceived by someone through series of certain operational steps. Processis the result of reflection on some similar actions that become unity, without requiring any external stimulus. Objectis someone's consciousness towards certain process as a whole, thus he can perform a further action on that object. Schema of a mathematical concept is a set of actions, processes, objects and other schemas linked by some general principles to establish a framework in the mind of someone, which can be applied to a situation of problem that involves the concept.

Based on the explanation above, it is considered significant to develop a mathematical learning approach that creates an effective pedagogical didactic situation by accommodating various student's learning styles, but remains within the framework of the existing theory of learning mathematics. The general principle used to develop the learning approach in question is:

Someone would perceive properly an abstract idea that has not been known yet if the meaning contained in the idea is presented in a form that can provide a clear picture.

Visualization is a word that the meaning can be an ability and process in providing picture (to visualize), or the product of the process of visualizing something abstract or complex (Rosken&Rolka, 2006), and possible to facilitate in understanding something abstract. The question is how the stages of learning process using visualization applied by still following the framework of existing learning theory of APOS?

II. VISUALIZATION IN MATHEMATICS

As a starting point of the study on visualization in mathematics, we start from the definition of visualization by Rosken and Rolka (2006) that has been summarized from several experts' opinions and viewpoints. It is said that visualization is an ability, process and product of a creation and interpretation by using picture, image or diagram in someone's mind, or on paper or by means of technology aiming to describe or communicate information, to think or develop ideas previously unknown, or to provide further understanding.

Based on the above definition, there are three points of view related to visualization, namely visualization as ability, visualization as a process and visualization as a product. In the process of learning mathematics, the three points of view are related one another. The product of visualization can be a way for teacher to transfer abstract knowledge of mathematics to students.

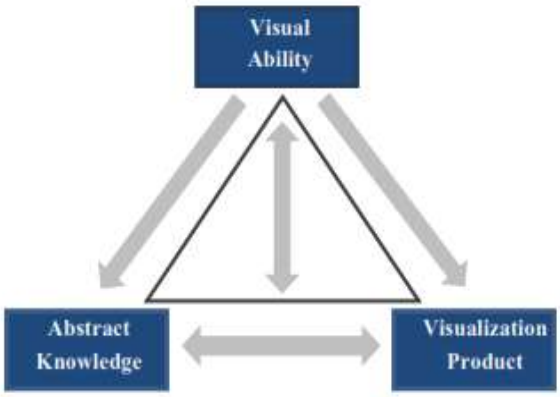


Figure 1. Triangle of Visualization Situation

Visual ability is related to the process of visualizing abstract knowledge into something that is easily recognized by the senses of sight (product of visualization), so that it appears new understanding in understanding the abstract knowledge. The relation between abstract knowledge, visualization product, and visual ability can be visualized as in Figure 1. Based on the figure, it shows that the ability in visualizing an abstract mathematical knowledge is possible to grow and develop when performing the process in reciprocal relationship between abstract knowledge and visualization product.

III. THE ROLE OF VISUALIZATION IN LEARNING

To determine the role of visualization in learning mathematics can be viewed from the standpoint of the definition of visualization, namely as ability, process and product. The role of visualization as ability and process is closely related to the complexity of a problem. At the primary school level, visualization is important in solving problems. Ho (2010) identifies seven roles of visualization in solving mathematical problems, namely (a) to understand how the elements of the problem is related one to another; (b) to simplify the problem, in the sense that students are able to identify a simpler version of the problem they deal with; (c) to see the relationship with other related problems (i.e. previous problem solving experience); (d) to accommodate different learning styles of students; (e) as a substitute for calculation; (f) as a mean to check to rationality of the answers obtained; and (g) to transform problem into mathematical form.

At the level of tertiary education, Guzman (2002) identifies that the role of visualization is not much different from its role at the primary school level in terms of dealing with complicated mathematical problem, in which visualization can stimulate the emergence of a variety of problem-solving strategies, and can provide description of the relation between the objects as a whole. Visualization can also become a quick tool in communicating an idea. Kashefi et.al (2015) emphasizes on the importance of the application of visualization in the learning process of mathematics in the classroom in order to overcome the difficulties of students in mastering basic skills required when solving a problem.

The role of visualization is seen as a product, Bagni (1998) states that visualization has versatile role at high school level. It can be a starting point and essential travel companion in developing learning materials. In addition, visualization can be one of the very beneficial forms of presentation in learning abstract mathematical concepts or procedures gradually.

At the university level, Guzman (2002) conveyed the importance of visualization in various types of mathematical analysis activities. Visualization is a very useful tool in the context of early mathematicalization of a concept or procedure as well as in the learning process when introducing an abstract mathematical idea.

IV. VISUALIZATIN AS LEARNING APPROACH

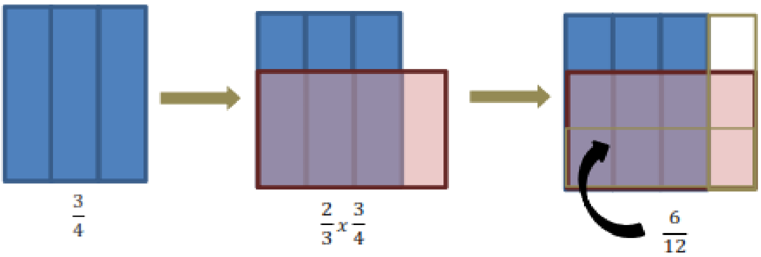
Based on the definition of visualization as previously explained, visualization can be viewed as a learning approach, in the sense of a way applied by teacher in achieving the learning objectives. As an approach, the knowledge and skill expected to obtain by students should be processed following the framework of the existing learning theories. The main idea in providing understanding of abstract mathematical knowledge is supported by the visual presentation. Then, students gradually internalize the presentation abstractly in their mind so that it will form a thinking framework ready for use when confronting problems in different contexts. Therefore, the stages in visualization approach follow the following sequence: visualization – representation – abstraction – schematization.

Definition of the term at every stage refers to some opinions relevant to mathematical context. The series of stages can be regarded as the operational form of APOS learning theory, which is begun with specialization in the stage of *action* in the form of students' interaction with the visualization product (didactic situation).

Stage 1: Visualization

Visualization is the first stage in the framework of APOS theory. This refers to the idea of Zaskis, Dubinsky and Dautermann (1996) who state that visualization is an action that creates strong connection between internal connection in the mind and something (i.e. object or event) through external media. See example 1 as the illustration of the definition.

Example 1: Visualization of $\frac{2}{3}x\frac{3}{4}$



The second process shows both fractions as external media in the sense that 2/3 of ¾. The areas that area shaded twice in the third process show the result of the multiplication of the two fractions. External media used in embedding the abstract idea is possible to stimulate the interaction among students, teacher and student (pedagogical situation), as well as the interaction between student and the mathematical content (didactic situation).

Stage 2 : Representation

Based on the definition of representation is psychology (Hwang et.al, 2007) and definition of *process* in APOS theory, the stage of representation in this learning approach means modeling process performed mathematically from several actions to make similar connection (i.e. visualization) into unity, without requiring external media. From this definition, someone is said to be performing representation of the multiplication result $\frac{3}{4}x\frac{1}{3}=\frac{3}{12}and\frac{2}{5}x\frac{3}{4}=\frac{6}{20}$
if he models it without external media. This result emerged from the process of reflection of students toward visualization of some of the results of multiplying two factions that have been done before. Indirect intervention can be done by teacher when students have not found the pattern of the result they obtained.

Stage 3 : Abstraction

Wells (2002, p.17) defines abstraction as follow:
An abstraction of a concept C is a concept C_ that includes all instances of C and that is constructed by taking as axioms certain statements that are true of all instances of C.
Example I: The concept of group is an abstraction of the concept of the set of all symmetries of an object. The group axioms are all true statements about symmetries when the binary operation is taken to be composition of symmetries.

Based on the notion of representation that has been formulated in the previous stage and definition of abstraction above, thus abstraction is a process of reconstructing a statement, which is true for all model resulted from the representation. Abstraction in the multiplication of two fractions is based on mathematical modeling process $\frac{3}{4} x \frac{1}{3} = \frac{3}{12}$ and $\frac{2}{5} x \frac{3}{4} = \frac{6}{20}$ without external media, then one realizes that if he has two fractions of $\frac{a}{b}$ and $\frac{c}{d}$, the product of the two fractions will also be a fraction in which the numerator is obtained by multiplying the numerator of the first fraction with the numerator of the second fraction, and the denominator is derived from multiplying the denominator of the first fraction with the denominator of the second fraction. Therefore, the product is $\frac{ac}{bd}$.

Referring to the definition of object in APOS theory, one can carry out further action toward the result of abstraction, such as: how to calculate the multiplication of $1\frac{2}{3}\ x\ 2\frac{3}{4}$, or how much is 50% of $3\frac{2}{5}$? Teacher can apply scaffoldingtechnique so that students are able to provide the further action.

Stage 4 : Schematization

The definition of schematization refers to the definition of *schema* in APOS theory by using notions of representation and abstraction in the previous stage. Thus, schematization is a process of establishing a framework of relation between several representations, abstractions, and possibly other frameworks, by using some general principles so that one can apply the framework on the problems involving certain concept. When someone deals with problem that involves multiplication of two fractions as follow:

Example 2 : $\left(\frac{1}{3}+\frac{2}{3}\right)\left(\frac{3}{4}-\frac{1}{2}\right) = ...$ and $\left(2\frac{1}{3}\right)\left(1\frac{1}{4}-\frac{1}{2}\right) = ...$

then there are some of the results of abstraction and thinking framework in each problem, which is the abstraction of addition/subtraction of two fractions either with the same or different denominator, and mixed fraction. If the thinking framework of multiplying two fractions has been formed in someone's mind, then any problems he deals with will be directed to

$$\left(\frac{1}{3}+\frac{2}{3}\right)\left(\frac{3}{4}-\frac{1}{2}\right) = \frac{3}{3} x \frac{1}{4} = \frac{3}{12}$$

$$\left(2\frac{1}{3}\right)\left(1\frac{1}{4}-\frac{1}{2}\right) = \frac{7}{3} x \left(\frac{5}{4}-\frac{1}{2}\right) = \frac{7}{3} x \frac{3}{4} = \frac{21}{12}$$

the forming of two fractions $\frac{a}{b}$ and $\frac{c}{d}$ as shown in the second step in the solution of the first problem, and the third step in the solution of the second problem.

V. CONCLUSION

Visualization is an alternative approach to learning that allows the creation of a didactic situation (i.e. interaction between students and the mathematical content), pedagogical situation (i.e. interaction between student and student, and student and teacher), and pedagogical didactic situation (i.e. interaction among student, teacher and mathematical content). Different learning styles of student are possible to accommodate in this approach, either those whose study is started from *sensory* and *visual* with learning process through visualization product, or *inductive* and *active* with learning through the activity of manipulating visualization product for some specific examples, as well as those whose study is started from *sequential* with gradual learning from the simple to the complex in a new context through the series of: visualization – representation – abstraction – schematization.

REFERENCES

[1]. Bagni, G.T. (1998). Visualization and Didactics of Mathematics in High School: an Experimental Research. *ScientiaPaedagogicaExperimentalis XXXV*, pp. 161-180.

[2]. Dubinsky, E. and M.A. McDonald, M.A. (2001) "APOS: A constructivist theory of learning in undergraduate mathematics education research" , In The Teaching and Learning of Mathematics at University Level: An ICMI Study, edited by D. Holton (Kluwer Academic Publishers, Dordrecht), pp. 275-282.

[3]. Felder, R.M. and Silverman,L.K. (1998). Learning and teaching styles in engineering education. *Engineering Education 78*, pp. 674-681

[4]. Freudenthal, H. (2002). Revisiting Mathematical Education China Lectures (Kluwer Academic Publishers, New York, 2002).

[5]. Guzman, M. (2001). The Role of Visualization in The Teaching and Learning of Mathematical Analysis. *Proceedings of the International Conference on the Teaching of Mathematics* (*at the Undergraduate Level*). Greece.

[6]. Ho, S.Y. (2010). Seeing the Value of Visualization.available at:http://singteach.nie. edu.sg/issue22-mathed/.

[7]. Hwang, W.Y. (2007). Multiple representation skills and creativity effects on mathematical problem solving using a multimedia whiteboard system.*Educational Technology & Society*, 10, pp. 191-212.

[8]. Kashefi, M. (2015). Visualisation in mathematics problem solving meta-analysis research.*Proceeding of the International Conference on Social Science Research*. Kuala Lumpur.

[9]. Rosken, B and Rolka, K. (2006). A Picture is worth a 1000 words – The role of visualization in mathematics learning. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* 4, pp. 457-464.

[10]. Suryadi, D. (2005). “PenggunaanpendekatanpembelajarantidaklangsungsertapendekatangabunganlangsungdantidaklangsungdalamrangkaMeningkatkankemampuanberpikirmatematikatingkattinggisiswa SLTP”, Ph.D. thesis, Indonesian University of Education.

[11]. Zazkis, R., Dubinsky, E. and Dautermann, J. (1996). Coordinating visual and analytic strategies: a study of students’ understanding. *Journal for Research in Mathematics Educations*, 27, pp. 435-437.

[12]. Walker, I. And Crogan, M. (1998). Academic performance, prejudice and the jigsaw classroom: new pieces to the puzzle. *Journal of Community and Applied Social Psychology*, 8, pp. 381 – 393.

[13]. Wells, C. (2002). A handbook of mathematical discourse.available at: http://www-math.cwru.edu/cfw2/ abouthbk.htm.